THE PREDICTIVE NATURE OF ALGEBRAIC ARITHMETIC FOR YOUNG LEARNERS

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The present study revalidated a measurement model describing the nature of early number sense. Number sense was shown to be composed of elementary number sense, conventional arithmetic and algebraic arithmetic. Algebraic arithmetic was conceptualized as synthesis of number patterns, restrictions and functions. Two hundred and four 1st grade students were individually tested on four different occasions. Data analysis suggested that elementary number sense follows a logarithmic growth, while conventional arithmetic and algebraic arithmetic adopt a linear growth rate until the third measurement and then they accelerate. Analysis showed that the growth of algebraic arithmetic directly predicts students' mathematics achievement in second grade and the growth of conventional arithmetic and indirectly the growth of elementary number sense.

INTRODUCTION

Researchers and organizations have documented the importance of enhancing students' early number sense (National Council of Teachers of Mathematics, 2000; National Mathematics Advisory Panel, 2008; Pittalis, Pitta-Pantazi & Christou, 2013). The development of students' number sense is considered as an important outcome and key ingredient of school curricula and a foundation for developing formal mathematical concepts and skills in elementary school (Yang, Li & Lin, 2007). Research findings support that number sense is a powerful predictor of mathematics outcomes and a vital prerequisite to success in mathematics (Malofeeva, Day, Saco, Young, & Ciancio, 2004).

The present study builds on previous studies asserting that early number sense consists of three distinct, but interrelated components (Pittalis, Pitta-Pantazi, & Christou, 2013). In particular, it was theoretically established and empirically validated that early number sense is a general theoretical construct that consists of three components (a) elementary number sense, (b) conventional arithmetic and (c) algebraic arithmetic. It was suggested that elementary number sense is comprised of key elements of numbers sense (see Jordan, et al., 2006), such as counting and number knowledge. Conventional arithmetic refers to story problems and number combinations that encompass number transformation situations. Finally, the proposed new component, algebraic arithmetic extends the two-dimensional model proposed by Jordan and her colleagues (2006) and incorporates number patterns and number equations.

In this study, we revalidated the structure of early number sense by encapsulating algebraic arithmetic component in a more comprehensive and systematic way, traced

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the development of number sense components and examined the way in which number sense growth factors relate to mathematics achievement. In particular the aims of the study were to: (a) validate the nature of early number sense components, (b) propose a growth model describing the development of number sense and (c) examine the relation between number sense growth factors and mathematics achievement.

THEORETICAL BACKGROUND

Research indicated that number sense is one of the most important concepts to be developed in early mathematics (Baroody, Eiland & Thompson, 2009). The quality of young children's number sense is a key predictor of later mathematical success; both in short and long term (Aunio & Niemivirta, 2010). For instance, research findings suggest that early number sense development contributes in learning more complex mathematics concepts; it promotes numerical fluency and is foundational to all aspects of early mathematical skills (Baroody, et al, 2009; Jordan et al., 2010). A number of research studies showed that inadequate development of number sense in early grades may be related to mathematical learning difficulties (Jordan et al, 2007). Moreover, Jordan and her colleagues (2010) showed that number sense is a powerful predictor of mathematics outcomes at the end of first grade and at the end of third grade, while number sense at the beginning and at the end of kindergarten was highly correlated with first grade mathematical achievement (Jordan et al., 2007). Locuniak and Jordan (2008) showed that kindergarten number sense was a strong predictor of calculation fluency in second grade, while Yang and her colleagues (2007) showed that the mathematics achievement of students in 5th grade was correlated with number sense performance. In addition, it is supported that students who enter school with strong number sense are more likely to benefit from teaching in the elementary grades and that the effect of weak number sense may be cumulative (Jordan et al., 2010).

A well-accepted and broad definition of number sense refers to a coherent understanding of what numbers mean, numerical relationships and the ability to handle daily life situations which involve numbers (Yang, 2005). Pittalis, Pitta-Pantazi, and Christou (2013), based on a synthesis of the literature, empirically validated a measurement model hypothesizing that number sense is a general second-order theoretical construct comprised of three first-order latent factors, namely (a) elementary number sense, (b) conventional arithmetic and (c) algebraic arithmetic. The proposed nature of number sense defines a more dynamic and flexible construct that may facilitate students' advancements and transition to a more abstract and relational system of thinking.

The foundation of algebraic arithmetic component of number sense lies on the research findings suggesting the introduction of students to algebraic reasoning at a much earlier age (Lins & Kaput, 2004). It should be noted that early algebraic reasoning conceptualizes algebra as a specific type of activity that builds on bridging arithmetic and algebra by promoting (a) understanding of the function of operations, (b) generalization and justification, (c) extension of the number system and (d) notation with meaning. These kinds of activities may contribute to the (a) transition of students from arithmetic towards algebra and (b) in the empowerment of arithmetic operations and computational fluency (Russell, Schifter, & Bastable, 2011).

The conceptualization of algebraic arithmetic component of number sense builds on Drijvers and his colleagues (2011) description of algebra as an amalgamation of (a) patterns and formulas, (b) restrictions and (c) functions. Examining the relations of these three stands with number sense, it can be conjectured that patterns, restrictions and functions may contribute in sustaining and further enhancing two major dimensions of number sense, namely, the relations among numbers and the conceptualization of the effect of operations of numbers. Moreover, this kind of activities may activate self-awareness mechanisms regarding the relations among numbers and promote self-reflection about the function and the properties of operations. Thus, in an attempt to provide a comprehensive and functional description of early number sense, we could suggest that "algebraic arithmetic" encompasses the development of a more sustainable and abstract understanding of the relations among numbers and of the effect of operations on numbers.

THE PRESENT STUDY

The purpose of the present study was to describe the nature of early number sense and explore the predictive validity of number sense growth factors on mathematical achievement. Specifically, the aims of the study were to (a) to revalidate the model proposed by the authors in PME36 suggesting that algebraic arithmetic is a component of early number sense, (b) to trace the development of six year old students' early number sense and (c) to investigate the relations among number sense latent growth factors and mathematics achievement.

In the present study, algebraic arithmetic component captures number patterns, functions and restrictions (equations and balance scale restrictions), as proposed by Drijvers and his colleagues (2011). The parameter of number patterns involves researching for regularity and patterns to recognize a common algebraic structure. The dimension of restrictions describes students' ability to find which value(s) of the unknown satisfies the required conditions in various situations; balance scale tasks or in more formal setting, such as equations. Finally, the function component involves students' ability to investigate arithmetic relations between quantities/variables.

Measures

The majority of the test items were adopted from the Curriculum Based Measurement (Fernstrom & Powell, 2007) and the rest ones were developed based on the theoretical considerations of the study. Six types of tasks were used to measure elementary number sense: (a) counting tasks, (b) number recognition, (c) quantity discrimination, (d) number knowledge, (e) enumeration, and (f) non-verbal calculation. In the counting tasks, students were asked to enumerate objects, in the number recognition tasks, students had to read numbers, in the quantity discrimination tasks students were asked to decide which was the largest number, in the number knowledge tasks, students were asked to find smaller and bigger numbers of a given one and in the non-verbal calculation tasks students had to add or delete objects in a given set so the number of objects corresponds to a given number. Conventional arithmetic factor was measured by three types of tasks: (a) story problems, (b) understanding of operations and (b) number combinations. In story problems tasks, students had to select the appropriate number sentence for a list of story problems. A set of new items were developed for understanding of operations in which students were presented with simple addition, subtraction and multiplication word problems and were asked to select out of 4 mathematical sentences, the one that fitted the problem. In number combinations tasks students had to find mentally the result of addition, subtraction, multiplication and division combinations. Finally, the proposed component algebraic arithmetic number sense was measured with four types of tasks: (a) number patterns, (b) restrictions-equations, (c) restrictions-balance scale and (d) functions. In number patterns tasks, students had to extend or complete number patterns, such as 5, 8, 11, … The ability to solve number patterns implies that a student can conceptualize the relations among numbers to fill in or extend a number pattern. For the assessment of students' abilities in number restrictions two types of tasks were used, number equations and balance scales. In the number equations, students were asked to complete the missing terms of equations, such as $3+5=4+\square$. The other restriction task appeared in the form of a balance scale. Students had to identify the value of two or three shapes which balanced in a balance scale with a given number. Finally, regarding student's abilities with functions, students were presented with function machines and a table which showed the input and output values. Students were requested to provide the input or output numbers which were missing. The task was an adaptation of a task presented by Drijvers and his colleagues (2011).

Mathematics achievement was measured with the Screening Assessment for Gifted Elementary and Middle School Students (SAGES-2). The SAGES-2 assesses aptitude and achievement in order to identify gifted students. We used the Mathematics subtest measures (K-3) which required not only recall but also understanding and application of mathematical ideas and concepts. In the present study, we used the mathematics score of SAGES-2 to measure students' mathematics achievement.

Participants, Procedure and Data Analysis

Two hundred and four first grade students were the subjects of the study. Students were assessed on the number sense measures four times during the period October to June (approximately one administration per two months) in the school year 2012-2013. During each measurement students were interviewed in two sessions of approximately 30 minutes each. Students had a time restriction for each type of task (one minute for the majority of tasks). Students were individually tested in all four occasions. The order of the parts was rotated in the four time series. The SAGES-2 test was administered a year later, in December 2013, when the subjects of the study were in the second grade.

The use of confirmatory factor analysis made sense because we wanted to examine the validity of an a priori model, based on past evidence and theory. CFA was conducted by using MPLUS, which is widely popular for its robust parameters (Muthén & Muthén, 2007). To trace the development of number sense components we used growth models. Growth models examine the development of individuals on one or more outcome variables over time. In order to evaluate model fit, three widely accepted fit indices were computed: The chi-square to its degree of freedom ratio (x2/df should be <2); the comparative fit index (CFI should be >.9); and the root mean-square error of approximation (RMSEA should be close to or lower than .08).

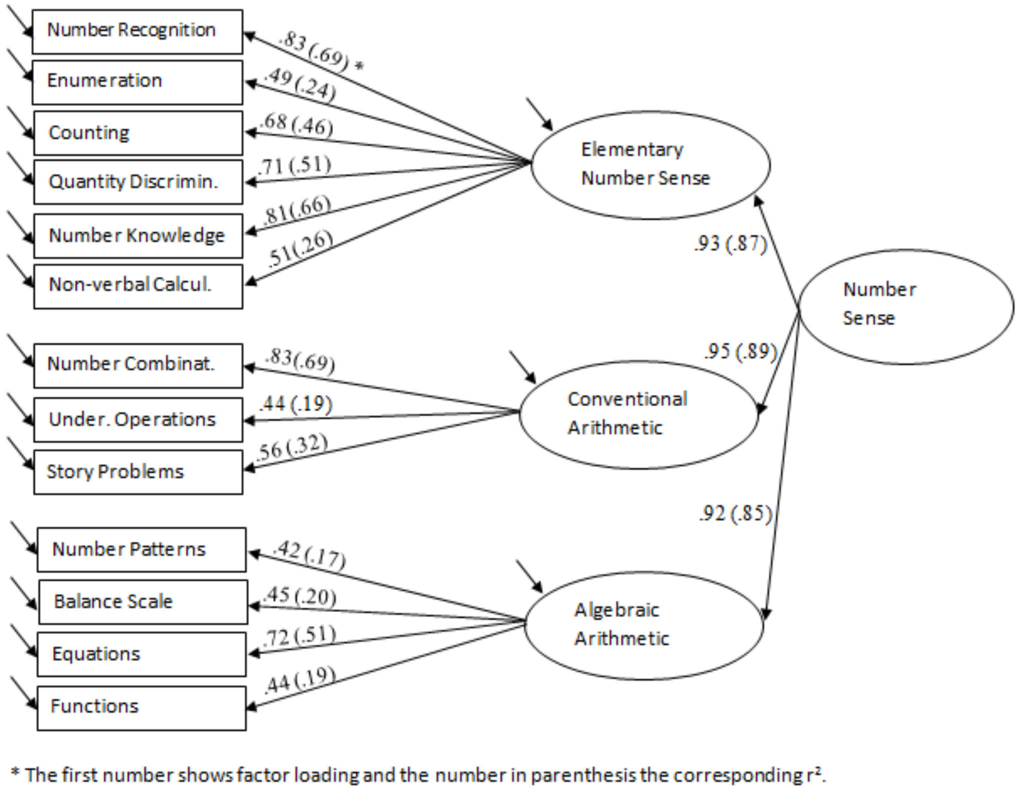

Figure 1: The nature of early number sense components.

RESULTS

The results of the analysis gave strong evidence to revalidate the construct validity of the hypothesized model describing the nature of early number sense (χ^2/df=1.60, CFI=.96, and RMSEA=.05). The results of the study showed that early number sense is a general, higher-order latent construct and might be described as a synthesis of three dimensions, namely, elementary number sense, conventional arithmetic and algebraic arithmetic. Figure 1 presents the standardized solution of the analysis and indicates that all factor loadings were statistically significant and most of them were rather large, ranging from .42 to .83 (see Figure 1). In addition, the factor loadings of the three first-order factors, elementary number sense, conventional arithmetic and algebraic arithmetic to the second-order factor, early number sense, were extremely high (.93, .95 and .92 respectively). Moreover, the predictive validity of the three components of number sense on the higher order early number sense factor was almost identical ($\underline{r}^2_{element}$. =.87, $\underline{r}^2_{algebraic}$ =.89, and $\underline{r}^2_{conventional}$=.85).

To examine the relation of number sense growth with mathematics achievement, first we examined the validity of alternative growth models. Longitudinal data in four time waves were used. The best fitting model (with the smallest AIC and BIC, χ^2/df=2.54, CFI=.98, and RMSEA=.08) was the one hypothesizing that elementary number sense follows a logarithmic growth rate while conventional arithmetic and algebraic arithmetic follow a linear growth rate until the third measurement and then they accelerate (see Figure 2). Thus, the results of the study showed that the growth of elementary number sense progressively is reduced, compared to the growth rate of the other components that increase. Elementary number sense had the largest mean latent slope (3.55), while conventional arithmetic mean latent slope was the second largest (3.28) and algebraic arithmetic mean latent slope was the smallest one (3.23).

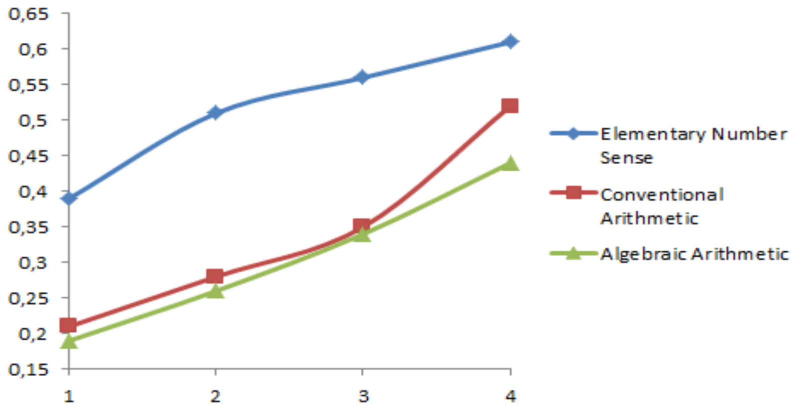

Figure 2: The development of number sense components.

Then, to examine the relation among number sense growth factors and mathematics achievement we tested alternative structural models. The adopted model (χ^2/df=2.18, CFI=.97, and RMSEA=.07) showed that the latent slope factor of the algebraic arithmetic component is a strong predictor of mathematics achievement ($\underline{r}$=.64) and of the latent slope factor of conventional arithmetic ($\underline{r}$=.70). Figure 3 shows that the slope factor of algebraic arithmetic had also an indirect effect on the latent slope factor of elementary arithmetic ($\underline{r}$=.32), through the latent slope of conventional arithmetic. In addition, the latent slope factor of conventional arithmetic had a direct effect on the latent slope of elementary arithmetic ($\underline{r}$=.46). The analysis showed that the intercept and the slope factors of algebraic arithmetic had a very high positive correlation, indicating that a student with a higher initial value in algebraic arithmetic would have a higher growth. On the other hand, the intercept and the slope factors of elementary arithmetic had a moderate negative correlation, indicating possibly a ceiling effect.

DISCUSSION

The results of the study reaffirmed the model describing the nature of students' early number sense, suggesting that early number sense consists of elementary, conventional and algebraic arithmetic components. Algebraic arithmetic includes number patterns, restrictions and functions. The proposed component of algebraic arithmetic adopts Drijvers and his colleagues (2011) definition of algebra. Elementary number sense develops with a logarithmic rate, while conventional arithmetic and algebraic arithmetic develop with a constant rate and accelerate progressively.

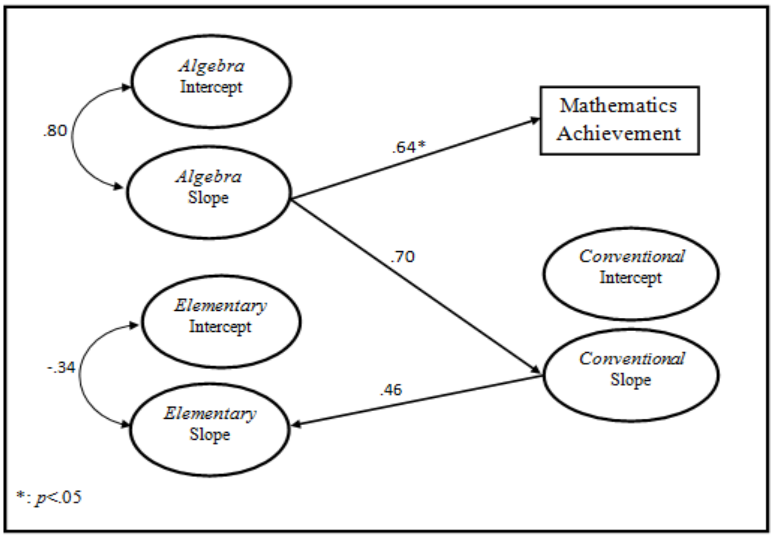

Figure 3: The relations among number sense and mathematics achievement.

The innovative aspect of the study lies on the exploration of the relation among the growth factors of number sense and mathematical achievement. The results of the study showed that students' algebraic arithmetic growth rate in the first grade predicts their mathematical achievement in the second grade. What is noteworthy is that neither the conventional arithmetic nor the elementary arithmetic could predict the second year's mathematical achievement. On the contrary, the growth rate of algebraic arithmetic proved to have a direct effect on the growth of conventional arithmetic and an indirect effect on the growth of elementary number sense through the growth of conventional arithmetic. This means, that algebraic arithmetic predicts conventional arithmetic and elementary arithmetic growth rates. Moreover, the intercept of algebraic arithmetic relates positively with its slope, suggesting that a student entering first grade with a good understanding of algebraic arithmetic will result in a significant growth rate of algebraic arithmetic. Thus, students entering primary school with a high value in algebraic arithmetic might exhibit high growth rate in algebraic arithmetic and consequently high growth rate in the two other components of number sense and high mathematics achievement in general. These findings highlight the dynamic nature of algebraic arithmetic and underlie the potential of integrating algebraic arithmetic situations in kindergarten. This algebraic arithmetic may be of the form of simple number patterns, restrictions and function activities appropriately developed for kindergarten children that enhance their understanding of the relations among numbers and flexibility with numbers.

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